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ABSTRACT

Three studies were conducted to assess the abstraction processes involved in the development of the ability to associate numerals with sets of the appropriate size (numeration). Experiment 1 examined the sequence of the ability to discriminate relative numerical magnitude, numerical equivalence, Arabic numerals, absolute size of a set, and numeration, with 90 children, aged 3 1/2 to 5 1/2. Quantity and numeral discriminations and the magnitude concept were significantly easier to learn than numeration, but the equivalence concept was not easier than numeration. Experiment 2 was designed to determine if the lack of familiarity with numerical symbols accounted for children's failure in the numeration task. The subjects, 63 children aged 3 1/2 to 4 years, 11 months were trained to associate either geometric shapes, realistic pictures, or numerals with sets of objects. Both the realistic figures and the geometric shapes were significantly more difficult to associate with the appropriate numerical set. Experiment 3 compared performance on numeration tasks with linear homogeneous, non-linear homogeneous, or linear heterogeneous sets, using 90 children, aged 4 - 5 1/2. Numeration was significantly more difficult with heterogeneous stimuli than with the other stimulus arrays. (Author/ST)

ABSTRACT

A series of three studies was conducted to assess the abstraction processes involved in the development of the ability to associate numerals with sets of the appropriate size (numeration). Experiment 1 examined the sequence of the development of the ability to discriminate relative numerical magnitude, numerical equivalence, Arabic numerals, absolute size of a set, and numeration, with 90 children, aged 3/6 - 5/5. Quantity and numeral discriminations and the magnitude concept were significantly easier to learn than numeration, but the equivalence concept was not easier than numeration. The results were interpreted in the terms of the young child's difficulties with the abstract meaning of numerals as opposed to the simpler perceptual discriminations involved in the other tasks.

Experiment 2 was designed to determine if the lack of familiarity with numerical symbols accounted for children's failure in the numeration task. The subjects, 63 children aged 3/6 - 4/11, were trained to associate either geometric shapes, realistic pictures, or numerals with sets of objects. Both the realistic figures and the geometric shapes were significantly more difficult to associate with the appropriate numerical set. Thus, the abstraction process itself, not the specific nature of numerical symbol, was assumed to account for the relative difficulty of the numeration task.

Experiment 3 compared performance on numeration tasks with linear homogeneous, non-linear homogeneous, or linear heterogeneous sets, using 90 children, aged 4/0 - 5/5. Numeration was significantly more

difficult with heterogeneous stimuli than with the other stimulus arrays. Thus, for the numeration task, the young child does not initially understand that the composition of the items of a set is irrelevant for numerical size.

THE DEVELOPMENT OF THE ABILITY TO UNDERSTAND NUMERICAL SYMBOLS

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Numeration, the association of an Arabic numeral with a set of the appropriate size, is, by its nature, an abstract process in which an arbitrary symbol is used to represent a fixed numerical quantity. To the extent that numeration involves symbolic processes, its development in the young child can be expected to occur later than simpler quantitative concepts, such as the perception of numerical equality and difference, which depend more on the ability to judge numerical size than on abstract functioning.

Relatively few studies of young children's quantitative concept development have addressed themselves directly to this issue. D'Mello and Willemsen (1969) found that children could match arrays on the basis of number before they could correctly associate a numerical symbol with an absolute quantity. Using five year old children, Wang, Resnick, and Boozer (1971) found a relationship between the ability to use numerals and counting. The first study in the present series tested the hypothesis that, because of its abstract nature, a task involving numeration ability would be more difficult than tasks involving the basic quantity perceptions of relative magnitude or numerical equivalence. To insure that inability to use numerical symbols was not a result of a failure to discriminate either the specific Arabic numbers or the absolute size of the sets, tasks were included which measured these abilities of the child.

To determine if the lack of familiarity with numerical symbols accounted for children's failure in the numeration task, the second study in this series trained children to associate numerals, geometric shapes, or realistic pictures with sets of one, two or three objects. Because the shapes and pictures were assumed to be easily labelled and less perceptually confusing than numerals, it was expected that the pictures or shapes would be more easily associated with the appropriate size set than numbers.

Previous research (e.g.,Siegel, 1972, 1973; Wohlwill, 1960; Zimiles, 1966) has indicated that certain perceptual factors, other than numerical size, influence a child's perception of numerosity. Heterogeneity of the constituents of a set and variations in the spatial arrangement of the set have been found to influence a child's perception of numerosity. The third study in this series compared the ability of young children to associate numerical symbols to homogeneous groups of objects in a linear array with numeration in regard to spatially irregular, homogeneous groups, or to linear, heterogeneous arrays. Heterogeneity and non-linear spatial arrangement were expected to make the numeration task more difficult.

EPXERIMENT 1: THE RELATIONSHIP OF NUMERATION ABILITY TO QUANTITY CONCEPTS

Method

Subjects

The subjects were 90 children from nursery schools in Hamilton, Ontario. For the purposes of data analysis, they were divided into the following age groups: 3/6 - 3/11, 9 girls and 8 boys; 4/0 - 4/5, 8 boys and 8 girls; 4/6 - 4/11, 19 boys and 13 girls; and 5/0 - 5/5, 12 boys and 13 girls. The nursery schools used in this study served both lower and middle socioeconomic class areas.

Tasks

All the subjects were administered five tasks: magnitude, equivalence, quantity discrimination, numeral discrimination, and numeration (1 - 3). A portion of the subjects were administered a sixth task, numeration (1 - 9), if they passed numeration (1 - 3). One of these tasks, the magnitude discrimination, was administered on a discrimination learning apparatus in which the stimuli were presented as slides and the child responded by pressing a button associated with either one of the pictures which appeared on the screen. The other four tasks were administered on a Behavioral Controls 400-SR programmed learning apparatus. There were four response panels covered with clear plastic press panels. The child responded by pressing the panel over the stimulus of his choice. The correct response was rewarded by a small candy. For all the tasks, a non-correction procedure was used and the position of the correct alternative varied from trial to trial.

Magnitude

The stimuli for this task were 50 sets of pictures, each stimulus consisting of two sets of dots. The number of dots in each set ranged from one to nine. The child was reinforced for choosing the set with the greater or, for counterbalancing, fewer number of dots. The child was instructed to press the button under the picture with the bigger (smaller) number of dots.

Equivalence

The stimuli for this task were 50 sets of samples for four alternatives; only one of the alternatives was identical in number to the sample. The size of the sample and the alternatives ranged between one and nine dots. The child was reinforced for choosing the alternative with the same number of dots as the sample. The child was instructed to press the panel over the picture with the same number of dots as the sample.

Quantity Discrimination

The stimuli for this task were 90 sets of three alternatives, each consisting of one, two, and three dots. For the first 30 trials, or until criterion which was 9 out of 10 consecutive correct responses, the child was reinforced for choosing one dot. For the next 30 trials, or until criterion, the reinforcement was for two dots, and for the last 30 trials, or until criterion, the reinforcement was for three dots. The child was instructed to press the panel over the picture with one (two, three) dots.

Numeral Discrimination

The task was similar to the quantity discrimination but the stimuli on each trial were the Arabic numerals 1, 2, and 3. The child was reinforced for each of these in turn, using the same system of 30 trial segments as in the quantity discrimination task. The child was instructed to press the panel over the picture with the number 1 (2, 3).

Numeration (1 - 3)

The stimuli were 50 sets of stimuli for a sample and four alternatives. The samples on each trial were the numerals 1, 2 and 3. The alternatives were sets of dots representing the numbers one through nine. The child was instructed to press the panel over the picture that had the number of dots that "went with the number" in the sample. If a child passed this task, he was given a numeration task in which the samples were the numerals 1 - 9. This task, administered only to children who passed numeration 1 - 3, was called numeration (1 - 9).

Procedure

Criterion for all tasks were 9 out of 10 consecutive correct responses. The children were tested individually and were administered the tasks in the following order: magnitude, equivalence, quantity discrimination, numeral discrimination, and numeration. In general, the testing was done in two consecutive sessions. The first session included both a rapport building period with the experimenter and the magnitude task and the second session included the remainder of the tasks. If the child appeared to be bored or restless, the testing was divided into three sessions on consecutive school days.

Results

Figure 1 shows the percentage of the subjects of each age group passing each task. A child was considered to have passed the

Insert Figure 1 about here

quantity and numeral discriminations if he reached criterion on all three parts of the task. Cochran $\underline{Q}$s (Siegel, 1956) were calculated for each age group to determine if there were significant differences among these percentages. The $\underline{Q}$ values were as follows: 3/6 - 3/11, $\underline{Q}$ (4) = 20.78, $\underline{p} < .001$; 4/0 - 4/5, $\underline{Q}$ (4) = 20.46, $\underline{p} < .001$; 4/6 - 4/11, $\underline{Q}$ (4) = 26.24, $\underline{p} < .0001$; 5/0 - 5/5, $\underline{Q}$ (4) = 6.35, $\underline{p} < .10$. Thus there was a significant difference among the five tasks at all but the oldest age group. The McNemar test (Siegel, 1956) was used to make individual comparisons between tasks for each age group, except the oldest where no comparisons were made because of the lack of overall significance. The magnitude task was significantly easier than the numeration task for the three youngest age groups, 3/6 - 3/11, $\underline{x}^2$ (1) = 6.75, $\underline{p} < .02$; 4/0 - 4/5, $\underline{x}^2$ (1) = 4.9, $\underline{p} < .05$; 4/6 - 4/11, $\underline{x}^2$ (1) = 9.91, $\underline{p} < .01$. The equivalence task was not significantly easier than the numeration task for any age group. There were no significant differences between the equivalence task and the more difficult numeration task (1 - 9), except in the oldest group 5/0 - 5/5, in which equivalence was significantly easier than numeration (1 - 9) $\underline{p} < .031$ (binomial expansion). It should be noted, however, that this represents a subsample (64.4%) of the total group of subjects because only children who passed the numeration (1 - 3) task were given the numeration (1 - 9) task.

For the three younger age groups the quantity discrimination was significantly easier than the numeration task: 3/6 - 3/11, $\underline{p} < .004$ (binomial expansion); 4/0 - 4/5, $\underline{p} < .008$ (binomial expansion); 4/6 - 4/11, $\underline{x}^2(1) = 8.10$, $\underline{p} < .01$. The numeral discrimination was also significantly easier than the numeration task: 3/6 - 3/11, $\underline{p} < .03$ (binomial expansion); 4/0 - 4/5, $\underline{p} < .002$ (binomial expansion); 4/6 - 4/11, $\underline{x}^2$ (1) = 4.90, $\underline{p} < .02$.

Discussion

While the numeration task was more difficult than the discrimination of relative magnitude, numerals, and quantities, it was not more difficult than the equivalence concept. The numeration (1 - 3) task used small set sizes for the correct response while the magnitude and equivalence concepts used a wider range of set sizes (1 - 9). The failure to find a difference between numeration (1 - 9) and the equivalence task, except in one age group, is evidence for the irrelevancy of set size with these particular tasks.

The discrepancy between the learning of the magnitude concept and the equivalence and numeration concept may be the result of a different kind of variable. Brainerd (1973) has suggested that there are two kinds of quantitative concepts which develop sequentially in the young child. Initially, the young child acquires concepts which involve the ordering of quantities and only later, concepts of correspondence or cardination which involve the understanding of a particular set size as a member of a number class. According to Brainerd's analysis, the magnitude task used in this study represents an order concept and the numeration and equivalence concepts are variations of a cardination concept. The findings of this present study, while not specifically designed to test Brainerd's thesis, indicate that order concepts may develop before certain kinds of class concepts.

The relative case with which the children solved the quantity and numeral discrimination indicates that although they could differentiate the symbols and the absolute quantities, the difficulty of numeration appeared to be the matching of numerals with sets. The quantity and numeral discriminations involved the simultaneous discrimination among numerals, while the numeration involved the successive discrimination among numerals. In the numeration task, it appeared to us that some of the children had difficulty in naming and differentiating the numerals.

The second study in this series tested the hypothesis that failure in the numeration task was, at least partially, a result of confusion about the number symbols and that more familiar and easily labelled objects might make the process of symbolizing less difficult for the child.

EXPERIMENT 2: THE ROLE OF NUMERICAL SYMBOLS IN NUMERATION TASKS

To determine whether or not the difficulty with the numeration was associated with the nature of numerals, or the abstraction process itself, the children were trained to associate different kinds of non-numerical symbols with number sets. Stimuli were chosen which were easily labelled and discriminable from each other. It was expected that pictures of familiar objects or geometric shapes would be easier to associate with number sets than numerals.

Method

Subjects

The subjects were 63 children ranging in age from 3/6 - 4/11 from nursery schools in Hamilton, Ontario. The nursery schools were located in both middle and lower class socioeconomic areas. There were 21 children at each of 3 age levels; 3/6 - 3/11, 12 boys and 9 girls; 4/0 - 4/5, 14 boys and 7 girls; 4/6 - 4/11, 11 boys and 10 girls. Within each age group there were three subgroups each composed of seven children; each one-third of the subjects at each age level was administered a different order of the tasks in a Latin-square design.

Tasks

Three tasks were used: numeration, pictures, and shapes. Each task consisted of 60 trials. The numeration was identical to that used in Experiment 1. For the pictures task, the sample was either a blue house, a red apple, or a yellow cat. The alternative were groups of one to nine dots. The child was reinforced for choosing one dot when the sample was a blue house, two dots when it was a red apple, and three dots when it was a yellow cat. For the shapes task, the samples were a red triangle, representing one; a blue circle, representing two; and a green square, representing three.

Procedure

Each child was tested individually. The criterion for all tasks was 9 out of 10 consecutive correct responses. If the child failed to reach criterion in 60 trials, the task was terminated.

Results

The means for the three age groups and three tasks are shown

Insert Table 1 about here

in Table 1. A Latin-square analysis of these data indicated significant effects of age ($\underline{F}$ (2,53) = 13.49 , $\underline{p}$ < .001) and task ($\underline{F}$ (2,116) = 53.003, $\underline{p}$ < .001) and a significant Task x Grade interaction ($\underline{F}$ (4,116) = 3.04, $\underline{p}$ < .025). At all ages the numbers task was significantly easier than either the pictures or the shapes task. There were no significant differences between the pictures and shapes task. There were no significant differences between the scores of boys and girls on any task (t-test).

Discussion

The results of this study indicate that the abstraction process involved in associating a symbol with a group of a particular size, rather than the specific nature of the symbol, accounts for the difficulty of the numeration task. Numbers, which are less easily labelled and more confusing than simple pictures and shapes, were no more difficult than the latter. While the discrimination among numerals may have been initially difficult for the children, with practice and feedback they could learn to associate numerals with sets more easily than they could learn the meaning of novel symbols. The relative difficulty of the shapes and pictures tasks indicates that while the children may have been unable to solve the numeration task on the required number of trials, some learning of number-numeral correspondence had taken place. Thus, their poorer performance on the shapes and pictures task was probably a result of negative transfer to these novel symbols.

EXPERIMENT 3: HETEROGENEITY AND SPATIAL FACTORS AS DETERMINANTS OF NUMERICAL ABILITY

The purpose of this study was to assess the ability of the child to recognize heterogeneity and spatial arrangement as irrelevant to numerical size. It was expected that the child would be less accurate in judging numerical size when the elements of the set were not identical to one an other, or when they were arranged in a non-linear array. Thus, the presence of these factors should increase the difficulty of the numeration task. To test this hypothesis, the children were administered tasks in which they were required to associate linear homogeneous, non-linear homogeneous, or linear heterogeneous arrays with numerals.

Method

Subjects

The subjects were 90 children, ranging in age from 4/0 - 5/5, from nursery sc ools in Hamilton, Ontario. The nursery schools were located in middle and lower socioeconomic areas. There were 30 children at each of three age levels; 4/0 - 4/5, 15 boys and 15 girls; 4/6 - 4/11, 15 boys and 15 girls; 5/0 - 5/5, 18 girls and 12 boys. At each age there were three subgroups, each composed of 10 children. Each subgroup was administered a different order of tasks in a Latin-square design.

Tasks

The children were administered three tasks: linear numeration, non-linear numeration, and heterogeneous numeration. The linear numeration task was identical to the task used in the previous studies. The non-linear numeration had stimuli similar to the linear numeration with the exception that the dots representing each number were placed in a random pattern within the space available.

The stimuli for the heterogeneous numeration were similar to the other numeration tasks but, instead of dots, the representations of numbers were geometric forms which varied, both within and between stimuli, in shape and color. The stimuli were in linear spatial arrays as in the numeration task, not in random arrangements as in the non-linear numeration task.

Procedure

The children were tested individually in one session. Criterion was 9 out of 10 consecutive correct responses . If criterion was not reached in 50 trials,the task was terminated.

Results

The means for each task at each age level are shown in Figure 2. A Latin-square analysis of these data indicated significant

Insert Figure 2 about here

effect of age (F (2,81) = 7.70, p <.004), task (F (2,162) = 11.25, p .001), order (F (2,162) = 14.59, p <.001), and a Task x Order interaction (F (2,162) = 8.26, p <.001). No other interactions were significant. Individual comparisons between tasks, using Duncan's multiple range test, indicated that the heterogeneous task was significantly more difficult than the other tasks at all age levels. The linear and non-linear tasks were not significantly different at any age.

The significant Task x Order interaction was the result of the heterogeneous task being significantly more difficult when administered first than when administered second or third. There were no order effects for the other tasks. Therefore, there appeared to be some positive transfer between heterogeneity and the other tasks. There were no significant differences between the performance of boys and girls on any task (t-test).

Discussion

The result of this study indicates that in tasks involving the association of numerals with sets, heterogeneity, but not spatial arrangement, interferes with the perception of numerical size. The abstraction ability involved in recognizing that a group of non-identical objects can represent a number is a significantly difficult concept for a child to learn, even with the small set sizes used in this study. Spatial arrangement does not appear to influence

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Footnote

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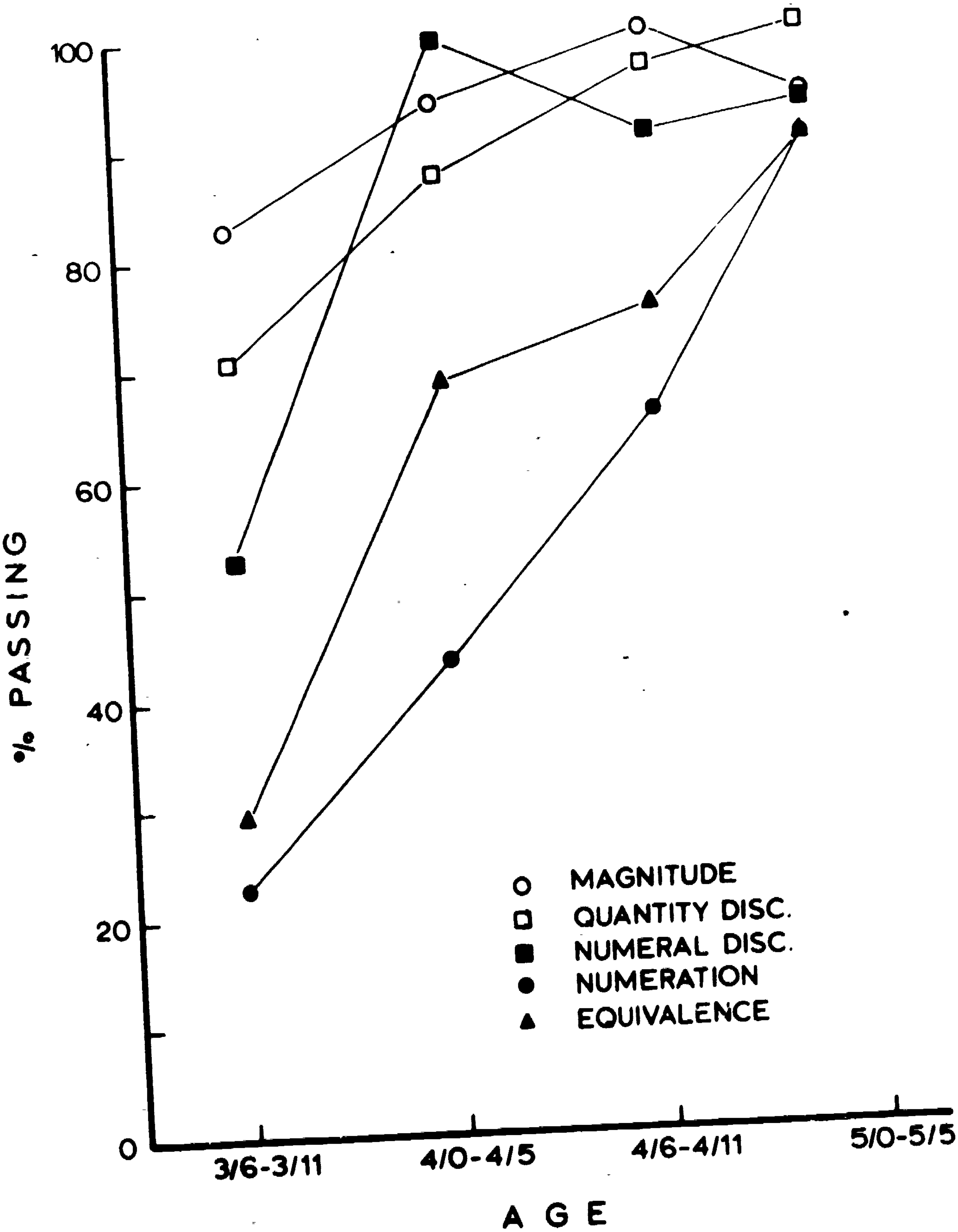
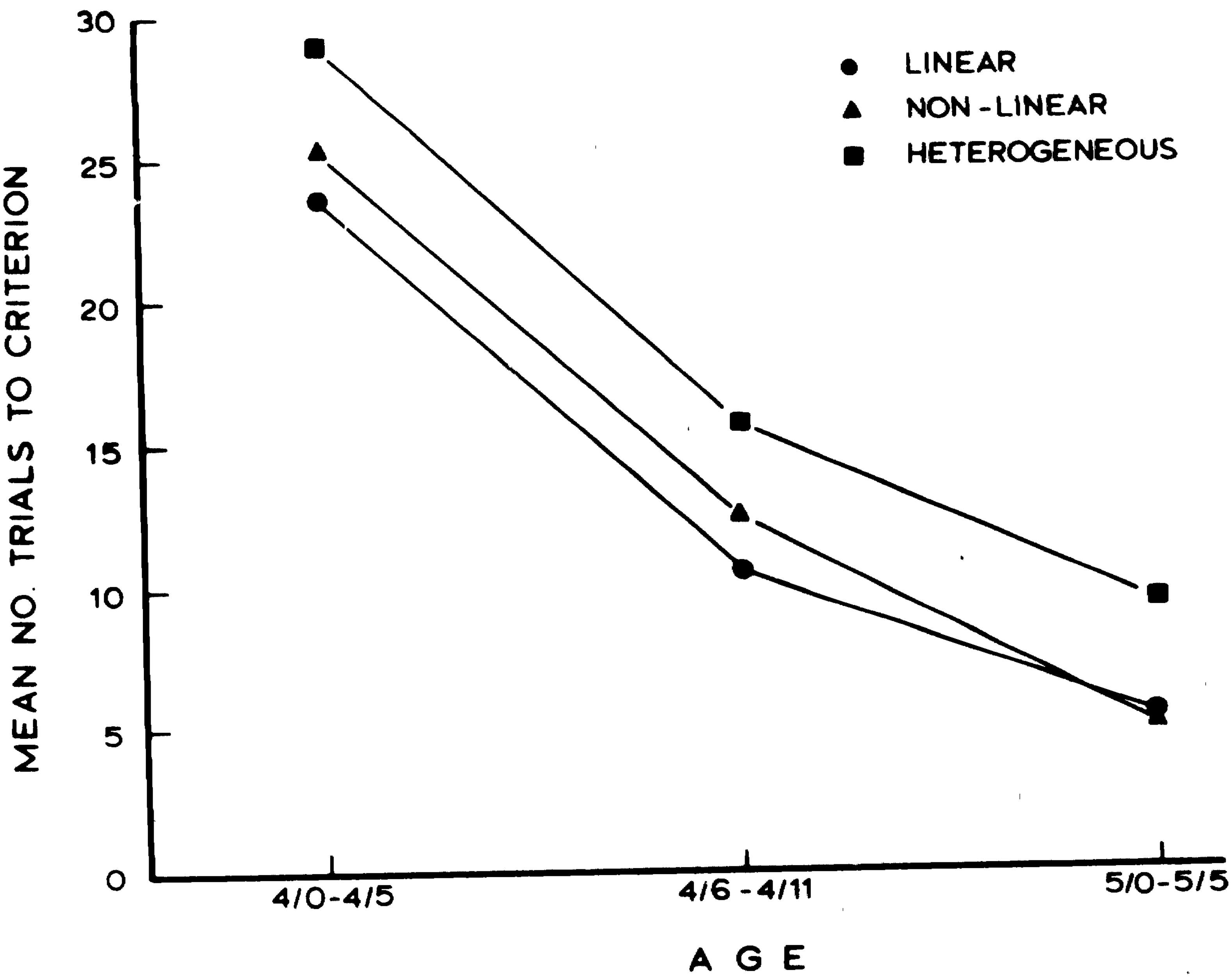
TABLE 1

Mean Number of Trials to Criterion

	TASK		
AGE	NUMBERS	PICTURES	SHAPES
3/6 - 3/11	44.19	59.05	56.90
4/0 - 4/5	24.05	47.71	51.71
4/6 - 4/11	17.48	45.67	52.38


100
80
60
40
20
0
% PASSING
MAGNITUDE
QUANTITY DISC.
NUMERAL DISC.
NUMERATION
EQUIVALENCE
3/6-3/11
4/0-4/5
4/6-4/11
5/0-5/5
AGE



MEAN NO. TRIALS TO CRITERION
30
25
20
15
10
5
0
LINEAR
NON-LINEAR
HETEROGENEOUS
4/0-4/5
4/6-4/11
5/0-5/5
A G E